1 General Information on Signals

1.1 Introduction

A signal is a physical representation of information that conveys that information from a source to a destination. It is the manifestation of a measurable physical phenomenon (e.g., voltage, current, acoustic pressure, temperature, displacement) as observed by a sensing or measurement device. Although many practical signals are electrical quantities (typically voltage, current, or electromagnetic field), signal theory is not restricted to any specific physical realization and applies equally to mechanical, acoustic, optical, magnetic, and other forms of signals. [1]

Signal theory comprises the mathematical tools used to model, represent, analyze, and manipulate signals and noise, whether produced by a source or modified by a system. Signal processing is the engineering discipline that, drawing on signal and information theory and enabled by electronics, computing, and applied physics, aims to acquire, transform, transmit, or interpret signals that carry information. It is therefore relevant to any application involving the generation, acquisition, transmission, storage, or exploitation of information.

1.2 Basic Definitions

1.2.1 Signal

A signal is a physical quantity that varies with one or more independent variables and is used to represent information. In many applications, the independent variable is time, and a continuous-time signal is denoted by $x(t)$. Examples include acoustic signals, optical signals, magnetic signals, and radio-frequency signals. In measurement and instrumentation, signals often arise from the controlled variation of measurable quantities such as voltage, current, force, temperature, pressure, or displacement.

1.2.2 Noise

Noise denotes any unwanted disturbance that degrades the detection, measurement, or interpretation of a signal. Typical sources include sensor noise, electromagnetic interference, quantization noise, thermal noise, and background disturbances. The distinction between "signal" and "noise" is context-dependent: a component that is noise for one task or user may be useful information for another.

1.2.3 Signal-to-noise ratio

The signal-to-noise ratio (SNR) quantifies the relative strength of a signal component compared with the noise component. When defined using power, it is the ratio of the signal power P_S to the noise power P_N, evaluated over the same measurement interval and bandwidth:

$$\mathrm{SNR} = \frac{P_S}{P_N}$$

It is often reported in decibels (dB) as

$$\mathrm{SNR}_{\mathrm{dB}} = 10\log_{10}\left(\frac{P_S}{P_N}\right). \tag{1.2.1}$$

If amplitudes are used instead of powers, the corresponding decibel expression uses $20\log_{10}(\cdot)$ because power is proportional to amplitude squared.

1.2.4 Signal processing

Signal processing is the discipline concerned with the acquisition, representation, transformation, and interpretation of signals in order to extract information, enhance signal quality, or enable efficient transmission and storage. It therefore spans applications such as measurement systems, communications, control, biomedical engineering, audio/image processing, and remote sensing. [2]

A typical measurement chain (Figure 1.2.1) can be described as follows: the physical phenomenon of interest is observed by a sensor (transducer) that converts it into an electrical signal (voltage or current). Noise and interference may be introduced at the sensor and in subsequent stages (e.g., analog front-end, transmission channel, and receiver). The received signal is then processed through operations such as filtering, detection, estimation, or reconstruction to obtain the desired information while mitigating the effects of noise.

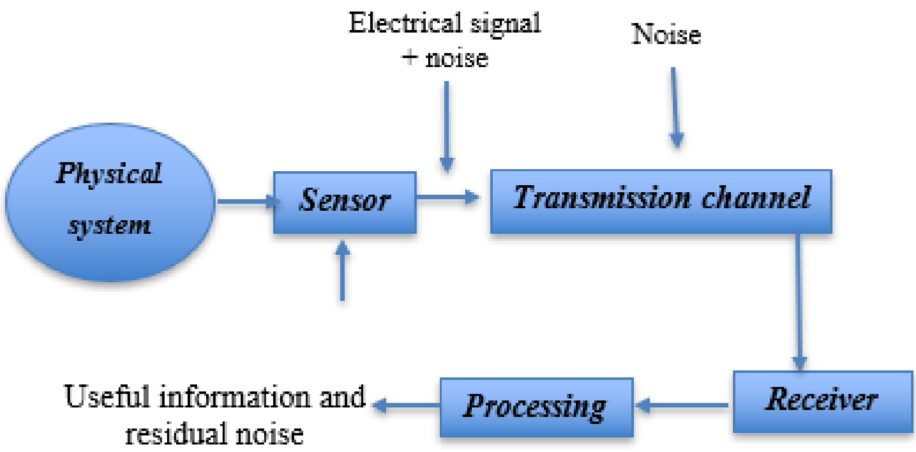

Figure 1.2.1. Signal transmission chain

1.3 Signal Classification

To support systematic analysis, signals may be classified according to several complementary criteria.

1.3.1 Phenomenological classification

This classification is based on the nature of the signal's temporal evolution. Two broad classes are distinguished:

- **Deterministic signals**: Signals whose values are completely specified by an explicit rule (e.g., a closed-form expression or a known algorithm). In principle, their evolution in time can be predicted exactly. This class includes, for example, periodic signals, transient (aperiodic) signals, and pseudo-random sequences generated by deterministic algorithms.
- **Random (stochastic) signals**: Signals whose instantaneous values cannot be predicted exactly. They are characterized statistically (e.g., via probability

distributions, moments, and correlation functions). If a random signal's statistical properties do not change with time, it is said to be stationary (more precisely, strict-sense stationary if all joint distributions are time-invariant, or wide-sense stationary if the mean is constant and the autocorrelation depends only on the time lag).

Examples of deterministic signals

A sinusoidal signal is a fundamental example of a periodic deterministic signal:

$$x(t) = A\sin\left(2\pi f t + \varphi\right)$$

where A is the amplitude, f the frequency (Hz), and φ the phase (rad).

Common examples of non-periodic deterministic signals include:

$$y(t) = e^{-at}, \quad t \geq 0 \ \ (a > 0),$$

$$z(t) = t,$$

$$w(t) = 1,$$

where *y(t)* is a (one-sided) decaying exponential, *z(t)* is a ramp, and *w(t)* is a constant (DC) signal.

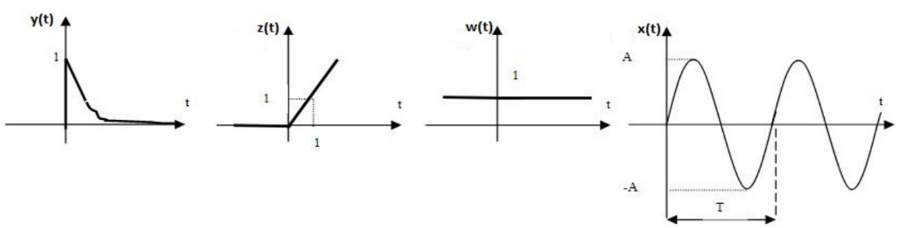

Figure 1.3.1 – Examples of deterministic signals

1.3.2 Energy-Based Classification

In this classification, we distinguish between signals that have finite energy and those that have finite average power (and hence infinite energy).

The energy of a continuous-time signal *x(t)* is defined as

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt. \tag{1.3.1}$$

A signal *x(t)* is said to be a finite-energy signal (or energy signal) if the above integral converges, i.e.,

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt < +\infty. \tag{1.3.2}$$

Finite-energy signals are also called square-integrable signals; mathematically, they belong to the space L2(R).

The average power of a continuous-time signal x(t) is defined as

$$P_x = \lim_{T \to +\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt. \tag{1.3.3}$$

A signal with finite, nonzero average power ($0<P_x<\infty$) is called a finite-power signal (or power signal). For such signals, the energy E_x is typically infinite.

Consequence. Any finite-energy signal has zero average power ($P_x=0$), whereas a finite-power signal necessarily has infinite energy (under mild regularity conditions) (Analogous definitions can be introduced for discrete-time signals *x*[n] by replacing the integral with a sum over n.)

1.3.3 Morphological Classification

Another useful classification is based on the “morphology” of the signal, i.e., on whether the independent variable (time) and the amplitude are continuous or discrete.

Let t denote time. A discrete-time signal is obtained, for example, by sampling a continuous-time signal at instants $t=kT$, where T is the sampling period and $k \in Z$.

Depending on the nature of time and amplitude, four types of signals are distinguished:

1. **Continuous-time, continuous-amplitude signal**: Often called a continuous-time (analog) signal. The independent variable t is continuous, and the signal can take any real (or complex) value in a continuum.
2. **Continuous-time, discrete-amplitude signal**: A continuous-time, quantised-amplitude signal. Time is continuous, but the amplitude can only take values from a discrete set (e.g., a finite set of levels).
3. **Discrete-time, continuous-amplitude signal**: Often called a discrete-time signal or sampled signal. The independent variable is discrete (e.g., $t=kT$), but the amplitude at each sample can take any value in a continuum.
4. **Discrete-time, discrete-amplitude signal**: Commonly referred to as a digital signal. Both time and amplitude are discrete: the signal is defined only at discrete instants and takes values from